

MENTAL ABILITIES: A FACTOR-ANALYTIC STUDY OF CHANGE¹

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About 7,500 students who had been tested as ninth-graders in Project TALENT were retested three years later as high school seniors. The primary purpose of this retesting was to study the growth and changes that occur during the high school years. As one part of this study, two 95-variable factor analyses were undertaken--one based on the boys and one on the girls.² Through the use of retest data many useful insights were obtained that wouldn't even have been guessed at if the factor analyses had been based on a single grade. This paper is concerned primarily with those findings that resulted from the use of retest data, rather than the ones that would have been obtained even if data were available for only one grade.

The 95 variables were grade 9 scores on 47 test variables, grade 12 scores on the same variables, and socioeconomic index. The 47 test variables all fell in the category of aptitude, ability, and achievement measures of one kind or another. (The names of the tests are listed in Table 1, along with the code numbers used to identify them in other tables and graphs.)

Procedure

Communality estimates were inserted in the diagonal of the matrix, and a principal factor solution was obtained. Since the widely used eigenvalue criterion for deciding how many factors to retain often eliminates some useful ones, particularly when N is large, the decision was based instead on an inspection of the distributions of off-diagonal residuals. On this basis, 17 principal factors were retained for the boys and 17 for the girls.

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²More detailed information about these factor analyses is presented in Chapter 6 of the full report on the retest study (Shaycoft, 1967).

Rotation. In rotating the factors, since it seemed desirable to avoid any preconceived notion about whether there should or shouldn't be a general factor, there was no potent reason for preferring varimax to quartimax. Any orthogonal solution that yielded clear readily interpretable factors would be satisfactory. The strategy settled upon was therefore to try quartimax first. If quartimax were to prove unsatisfactory varimax would be the next choice, and if this too failed the final tactic would be to resort to subjective rotation, maintaining orthogonality. As things turned out, this series of eventualities was exactly what occurred. The quartimax solution failed to meet the criterion of positive manifold; it produced several substantial bipolar factors. The normal varimax procedure eliminated this difficulty but did not produce a solution that had a clear interpretation. This made it necessary to resort to subjective rotations.

An example of one such rotation of two varimax factors is shown in Figure 1. The ten pairs of variables with substantial loadings are plotted; all except two are direct measures of language skills. The high loadings on varimax factor V_5 are on variables measuring language skills (reading, English, etc.) in grade 12 while variables measuring the same skills in grade 9 have low loadings. On factor V_{10} on the other hand, the grade 9 measures of language skills have substantially higher loadings than the same measures for grade 12. A factor which plays a major role in language skills in grade 9, only to shrink in importance by the time grade 12 is reached and be largely replaced by a brand new factor, is not a helpful explanatory concept. A 27° rotation of the axes, as shown in Figure 1, greatly improves the picture. One of the two new factors produced, F_4 , has about equal loadings on corresponding grade 9 and grade 12 variables; its high loadings are on the language skills variables. And the other new factor, F_6 , has high loadings for grade 12 on the language skills variables and essentially zero loadings for grade 9 on the same variables; it also has essentially zero loadings for both grades on other tests. (The complete set of pre-rotation and post-rotation loadings for the factors involved in this particular rotation is shown in Table 1.) Factor F_4 apparently represents the level of mastery reached in language skills as of grade 9,

and is independent of anything that happens during the high school years. This doesn't mean, of course, that scores on the English tests are unaffected during the period in high school. Quite the contrary! But to see what these effects are and how they operate we have to look to factor F_6 (Δ -ENG), which is probably a measure of growth in English skills. Factor F_6 is clearly related to change between grades 9 and 12 in the student's relative standing within the group in regard to mastery of English. But the important thing to note is that it represents a component completely unrelated to factor F_4 , which represents English ability in general. It may depend on the kind of formal classwork that is offered in English or it may be a function of interest or lack of it--or perhaps it is a combination of these. But whatever it is, it is apparent that a new factor of some sort is entering the picture, too late to affect any grade 9 scores much but in plenty of time to have a major effect on grade 12 scores related to mastery of the English language.

Final Set of Factors

The final sets of factors, for both males and females, are summarized in Table 2 and the detailed factor patterns are presented in Table 3. Table 2 indicates the nature of each factor and its source (varimax, test-specific, or subjectively determined rotation). (The concept of test-specific factors will be discussed later.) Table 2 also indicates the proportion of variance each factor accounts for, and shows the general similarity between the factors for males and the ones for females. It is apparent from this table that most of the more important factors are concentrated in the subjectively rotated factor category.

Factors Measuring Change. Factor F_7 , like factor F_6 , is a "change component" of the test variables. These two factors are particularly interesting because they probably would not have shown up in any kind of factor-analytic study that didn't involve testing at at least two periods several years apart. And if by some strange quirk these factors had shown up in a factor analysis based on a single testing (e.g., during

grade 12) what they represented would almost certainly have gone unrecognized since adequate data for interpreting them would have been lacking.

Factor F_7 , which showed up for boys but not for girls, bears about the same relation to factor F_1 (the general factor) that F_6 (Δ -ENG) bears to F_4 (ENGL). It represents that component of change in relative breadth of general information during the high school years that is independent of the student's status in this respect as a high school freshman. Variables with high loadings in this factor are not those in which twelfth-grade score is likely to be primarily a function of the level of achievement reached by grade 9, plus maturation.

Extraction of Test-Specific Doublet Factors. Table 4A shows the distributions of three categories of residuals after extraction of 17 principal factors. The three categories are: (1) diagonal terms, (2) off-diagonal terms representing the residual of the correlation between a grade 9 variable and the corresponding grade 12 variable, and (3) all other off-diagonal residual terms. Let's focus our attention on the second category--the off-diagonal residuals in the column headed "Corresponding." Not only did almost all of them turn out to be positive but they tended to be systematically higher than the other off-diagonal residuals. This fact, together with *a priori* expectations that linkages explaining variance unique to a particular test but common to the grade 9 and grade 12 scores obtained with this test would manifest themselves, suggested that each of these non-trivial residuals between corresponding grade 9 and grade 12 variables could legitimately be considered the basis of a doublet factor--a factor entering into only the two variables involved.

Thus we apparently had a situation in which subjective judgment should be allowed to supplement the principal factors through the addition of some of the "test-specific" factors revealed by the inspection of residuals. This meant the addition of new factors, orthogonal to the first 17 principal factors and to each other, but not themselves principal factors (and in most cases probably not even obtainable from the full set of principal factors by rotation).

Loadings for each of these new factors were defined as being 0 for all variables other than the grade 9 score and grade 12 score for the test

they referred to. For these two variables the loadings were indeterminate, the only restriction imposed being that their product had to equal the residual of their correlation. In the absence of any specific hypothesis to the contrary, equal loadings were assigned to the ninth-grade score and the twelfth-grade score. (These loadings were necessarily equal, of course, to the square root of the residual.)

It was decided to extract inter-grade doublet factors only for those tests for which the inter-grade residual correlation was at least $+.04$, so that its square root, used as the loading, would be at least $.20$. On this basis, 23 test-specific doublet factors were added for the males, and (strictly by coincidence) exactly the same number for the females. The loadings on these factors are shown in the next-to-last pair of columns on the third and fourth pages of Table 3.

The revised distribution of residuals is shown in Table 4B. It is quite apparent from a comparison of Tables 4A and 4B that extraction of the test-specific factors improves the factor solution materially. The mean of the off-diagonal correlation residuals for corresponding grade 9 and grade 12 variables is no longer markedly further from 0 than the mean for residuals of non-corresponding variables, nor is the standard deviation of the residuals any longer nearly twice as large for corresponding variables as for non-corresponding ones.

Because of the way the doublet factors were generated, they are necessarily orthogonal to all the other factors and they are in every sense just as "legitimate" as any of the other factors even though not derived from a rotation of principal factors.

Speed versus Accuracy. Factors 8a and 8b are rather remarkable; they show up in exactly the same tests, and with rather high loadings, but for different grades. Factor 8a has all its major loadings on the grade 9 scores and factor 8b on the grade 12 scores. The tests that these factors show up in are the ones that are highly speeded, and measure speed and accuracy in some sort of perceptual task (specifically the Object Inspection, Clerical Checking, and Table Reading tests, and to a lesser extent, Arithmetic Computation). Except for factors 8a and 8b there is no factor that links together the speeded tests and only those tests. Thus there appears to be no single factor that clearly represents a relatively

stable student characteristic corresponding to perceptual speed or accuracy or an amalgam of the two. What we have instead is two separate factors, one for grade 9 and one for grade 12; and because they are separate and totally uncorrelated it seems extremely doubtful that either of them represents any characteristics with a great deal of stability.

One possibility is that the factor represents the student's temporary set on the speed-versus-accuracy problem, and his approach to it. In any highly speeded test calling for performance on a task which the student can perform perfectly provided he devotes sufficient care to it, he may either devote an extravagant amount of time to each question in turn, sacrificing speed to accuracy, or he may zip through the test as fast as he can, sacrificing accuracy to speed, or he may operate on some basis that is a compromise between these two extremes. Which of these principles he operates on may depend in large part on his interpretation of what is important in the test or group of tests. The speed-versus-accuracy set under which he happens to be operating on a particular occasion may last long enough to affect the several tests given during the course of the day.

Some readers may wonder why factors 8a and 8b weren't rotated in such a way as to produce a sizable speed-and-accuracy-of-perception factor having substantial loadings on both the grade 9 and grade 12 speeded tests, since such a factor would clearly be readily interpretable. The answer is that if such a factor were forced into the pattern it would automatically bring with it as a partner an uninterpretable bipolar and therefore unacceptable factor. Figure 2 provides graphic evidence on this. In that figure, loadings of the speeded tests (and socioeconomic index) on factors 8a and 8b have been plotted. The two dotted lines (F_{8d} and F_{8c}) show one rotation of axes that was considered carefully and rejected. Factor 8d would have had at least one advantage: all the speeded tests would have had substantial loadings on it that were about equal for the grade 9 and grade 12 measures, while the unspeeded tests would have had loadings very close to zero. This would have been convenient but the other part of the package, factor 8c, would have had substantial positive loadings for the grade 12 speeded tests and substantial negative loadings

for their grade 9 counterparts, (or vice versa). There seems to be no simple and reasonable interpretation for a factor of this sort.

Perhaps the search for a large factor linking several different kinds of speeded tests is a search for something that just doesn't exist.

Effects of Basing Factor Analysis on Retest Data

The use of retest data as a basis for factor analysis has two general effects. It increases the number of common factors extracted and it changes and clarifies the interpretation of several of the factors.

Number of Common Factors Extracted. In the present factor analyses 40 common factors, all of them interpretable, were extracted for males and 40 for females. This is probably a much larger number than is usual, even for a large battery. Perhaps we can attribute the increase in part to methodological innovations such as the extraction of test-specific doublet factors to supplement the principal factors. But most of the increase in number of common factors is almost certainly due to the fact that the same tests were administered twice. That is what caused the doublet factors to exist. If the tests had been administered in grade 12 only, only about 11 or 12 common factors would have turned up--and even fewer if the single administration occurred in grade 9 since the "change factors" (Δ -ENG and Δ -INF) would not have appeared under those circumstances. Obviously if the testing had been confined to a single grade none of test-specific factors would have been extractable as common factors. The existence of these test-specific factors shows that it is not sound practice to eliminate tests from a battery merely because they lack substantial loadings on common factors. It cannot be assumed that all unique variance is necessarily unreliable variance. Factors representing unique variance within grade may possess considerable stability across grades.

Effect of Retest Data on Factor Interpretation. Some of the factors extracted would have been misinterpreted if they had been derived from a within-grade factor analysis instead of from retest data. Among these factors would be the ones now called Δ -ENG, Δ -INF, and SP-2 (or its partner, SP-1, if the within-grade factor analysis had been for grade 9

instead of grade 12). The Δ -ENG factor would almost certainly have been regarded as another factor of the same type as factor F_4 (ENGL), if, indeed, it hadn't been swallowed up in it and disappeared entirely. Similarly Δ -INF would either have been swallowed up in the first factor or would be regarded as merely a variant of it. The essential character of Δ -ENG and Δ -INF, as measures of change independent of initial status, would have gone unrecognized. Likewise SP-2, instead of being regarded as a rather puzzling and ambiguous phenomenon that might represent short-term set would confidently, but erroneously, be interpreted as representing a large and stable speed-and-accuracy-of-perception factor, presumably carrying across several grades. Thus it requires a retest situation to demonstrate that these factors lack grade-to-grade stability.

Factor Analysis of Retest Data

In conclusion, there are three ways in which use of retest data provides insights that would be unavailable in the usual factor analysis, based on a single administration of the battery:

1. It permits the extraction of test-specific common factors.
2. It reveals "change factors" that otherwise would not appear as separate factors--or if they did would almost certainly be misinterpreted.
3. It prevents misinterpretation and misunderstanding of some other common factors as well.

Of course retesting is not always feasible or desirable from an administrative viewpoint, but when retest data are available, much information can be gained by including both sets in the same factor analysis.

REFERENCE

Shaycoft, Marion F. *The high school years: Growth in cognitive skills.* (Interim report 3 to the U. S. Office of Education, Cooperative Research Project No. 3051.) Pittsburgh: Project TALENT Office, American Institutes for Research and Univer. of Pittsburgh, 1967.

Tables and Figures
to accompany

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TABLE 1. Factor loadings on a pair of varimax factors, and on the same pair after subjective rotation^a
(In factor analysis of Matrix 1A: Males)

		FACTOR LOADINGS							
		Varimax factors				Subjectively rotated factors			
		V ₁₀		V ₅		F ₄		F ₆	
		Gr. 9	Gr. 12	Gr. 9	Gr. 12	Gr. 9	Gr. 12	Gr. 9	Gr. 12
Information I									
R-102	Vocabulary I	.136	.068	.136	.148	.183	.128	.060	.101
R-103	Literature	.085	-.007	.135	.203	.137	.086	.081	.184
R-104	Music	.119	.102	.078	.117	.142	.144	.015	.058
R-105	Social Studies	.019	-.042	.120	.168	.072	.039	.098	.169
R-106	Mathematics	.087	-.045	.014	.100	.084	.005	-.027	.109
R-107	Physical Science	.014	-.025	.111	.102	.063	.024	.092	.102
R-108	Biological Science	.027	-.058	.143	.172	.089	.026	.115	.180
R-109	Scientific Attitude	.147	.079	.164	.143	.206	.135	.079	.092
R-110	Aeronautics and Space	.088	.021	.038	.044	.095	.039	-.006	.029
R-111	Electricity and Electronics	.065	.037	-.015	-.010	.051	.028	-.042	-.026
R-112	Mechanics	.146	.093	.083	.061	.168	.110	.008	.012
R-113	Farming	.089	.062	.112	.156	.130	.126	.059	.111
R-114	Home Economics	.036	.012	.048	.080	.054	.047	.027	.066
R-115	Sports	.076	.028	.057	.063	.094	.053	.016	.044
Information II									
R-131	Art	.067	.094	.159	.175	.132	.163	.112	.113
R-132	Law	-.006	-.004	.068	.100	.025	.042	.063	.091
R-133	Health	.051	.096	.134	.143	.106	.150	.096	.084
R-134	Engineering	.054	.134	.083	-.026	.085	.108	.049	-.084
R-139	Acct., Bus., Sales	-.062	.024	.045	.029	-.035	.035	.068	.015
R-140	Practical Knowledge	-.008	.009	.161	.130	.066	.067	.147	.111
R-142	Bible	.051	.139	.109	.116	.094	.177	.074	.040
R-145	Hunting	-.000	.027	.035	-.039	.015	.006	.031	-.047
R-146	Fishing	-.009	.060	.049	-.002	.014	.052	.048	-.029
R-147	Outdoor Activities (other)	.053	.196	.087	.052	.087	.199	.054	-.043
R-150	Theater; Ballet	.114	.182	.114	.139	.153	.225	.050	.041
R-162	Vocabulary II	.042	.108	.128	.170	.096	.173	.095	.102
R-212	Memory for Words	.136	.029	.045	.235	.142	.132	-.022	.196
R-220	Disguised Words	.302	.142	.191	.463	.356	.336	.033	.348
English									
R-231	Spelling	.343	.190	.209	.488	.400	.390	.030	.349
R-232	Capitalization	.465	-.009	.275	.737	.539	.327	.034	.661
R-233	Punctuation	.334	.138	.217	.520	.396	.359	.042	.401
R-234	English Usage	.437	.253	.220	.505	.490	.455	-.003	.335
R-235	Effective Expression	.392	.281	.166	.475	.424	.466	-.030	.296
R-240	Word Functions	.207	-.000	.124	.363	.240	.164	.016	.324
R-250	Reading Comprehension	.232	.035	.176	.464	.286	.242	.052	.398
R-260	Creativity	.239	.054	.088	.352	.253	.208	-.030	.290
R-270	Mechanical Reasoning	.184	.010	.021	.258	.173	.126	-.064	.226
R-281	Visualization in 2 Dimensions	.043	-.060	-.011	.079	.034	-.017	-.030	.098
R-282	Visualization in 3 Dimensions	.064	-.046	-.050	.159	.035	.032	-.074	.162
R-290	Abstract Reasoning	.169	.047	.053	.236	.175	.149	-.030	.189
Mathematics									
R-311	I. Arithmetic Reasoning	.156	.075	.075	.320	.173	.212	-.004	.251
R-312	II. Intro. h.s. Math.	.150	-.009	.073	.277	.167	.118	-.003	.250
R-333	III. Adv. h.s. Math.	.037	-.033	-.058	.173	.006	.049	-.069	.169
F-410	Arithmetic Computation	.133	.044	.047	.258	.140	.156	-.019	.209
F-420	Table Reading	.028	.059	.098	-.021	.069	.043	.074	-.045
F-430	Clerical Checking	.088	-.054	.117	.254	.131	.067	.064	.251
F-440	Object Inspection	-.002	.022	-.029	.079	-.014	.056	-.025	.060
P*801	Socioeconomic Index	.066		.028		.072		-.006	
Sum of squares		1.79		3.80		3.37		2.21	
Sum of squares for factor pair		5.59				5.58			

^aSee Figure #-1.

TABLE 2. Summary of common factors

Nature of factor	M A L E S			F E M A L E S												
	Factor #	Kind of factor*	Σb ²	Mnemonic	Factor #	Kind of factor*	Σb ²	Proportion of variance		Mnemonic						
								Common	Total							
1 General verbal	F-1	S	18.17	VERBL	F-1	S	20.65	.288	.191	VERBL	F-1	S	20.65	.350	.217	VERBL
2 Mathematical	F-2	S	10.27	MATH	F-2	S	2.04	.163	.108	MATH	F-2	S	2.04	.035	.021	MATH
3 Spatial	F-3	S	3.48	SPACE	F-3	S	4.51	.055	.037	SPACE	F-3	S	4.51	.077	.048	SPACE
4 English	F-4	S	3.37	ENGL	F-4a	V	4.19	.054	.035	ENGL	F-4a	V	4.19	.071	.044	ENG-A
5 Technical information	F-5	V	2.80	TECH	F-4b	S	2.41	.045	.029	TECH	F-4b	S	2.41	.041	.026	ENG-B
6 Gain in English skills	F-6	S	2.21	Δ-ENG	F-5	S	3.12	.035	.023	TECH	F-5	S	3.12	.053	.033	TECH
7 Gain in general info.	F-7	S	2.86	Δ-INF	F-6	S	2.06	.045	.030	Δ-ENG	F-6	S	2.06	.035	.022	Δ-ENG
8a Speed and timing: Grade 9	F-8a	V	1.95	SP-1	F-7	S	2.86	.031	.021	Δ-INF	F-7	S	2.86	.029	.018	SP-1
8b Speed and timing: Grade 12	F-8b	V	2.06	SP-2	F-8a	V	1.74	.033	.022	SP-1	F-8a	V	1.74	.040	.025	SP-2
9 Rural	F-9	S	2.15	RURAL	F-8b	V	2.06	.034	.023	SP-2	F-8b	V	2.36	.060	.037	RURAL
10 Fishing and Hunting	F-10	S	2.06	F & H	F-9	S	3.52	.033	.022	RURAL	F-9	S	3.52	.020	.012	FISH
11 Bible information	F-11	V	1.76	BIBLE	F-10a	V	1.16	.028	.019	F & H	F-10a	V	1.16	.017	.011	HUNT
12 Memorization	F-12	V	1.49	MEM	F-10b	S	1.02	.024	.016	BIBLE	F-10b	S	1.02	.034	.021	BIBLE
13 Common sense	F-13	V	1.26	SENSE	F-11	V	2.01	.020	.013	MEM	F-11	V	2.01	.024	.015	MEM
14 Arithmetic computation	F-14	V	1.62	COMP	F-12	V	1.40	.026	.017	SENSE	F-12	V	1.40	.024	.015	SENSE
15 Sports information	F-15	V	1.47	SPORT	F-13	V	1.41	.023	.017	COMP	F-13	V	1.41	.024	.015	SENSE
16 Home economics info.	F-16	V	1.35	HOME	T-115	T	.19	.023	.015	SPORT	T-115	T	.19	.003	.002	SPORT
17 Aero-space information	T-110	T	.12	AERO	T-114	T	.14	.021	.014	HOME	T-114	T	.14	.002	.001	HOME
18 Engineering information	T-134	T	.08	ENGIN	F-17	V	1.35	.002	.001	AERO	F-17	V	1.35	.023	.014	AERO
19-44 Other test-specific common factors					F-18	V	.96	.001	.001	ENGIN	F-18	V	.96	.016	.010	ENGIN
16 factors: both sexes																
5 factors: males only																
5 factors: females only																
"Total common variance"			95 Σ _{i=1} ² = 62.99				95 Σ _{i=1} ² = 58.99	1.000	.663				1.000	.621		
Unique variance			32.01				36.01		.337					.379		
"TOTAL VARIANCE"			95.00				95.00		1.000					1.000		

*V = Varimax S = Subjectively rotated T = Test-specific

TABLE 3. (second page)

Variable #	F E M A L E S																				
	F-1 VERBL		F-2 MATH		F-3 SPACE		F-4a ENG-A		F-4b ENG-B		F-5 TECH		F-6 Δ-ENG		F-8a SP-1		F-8b SP-2		F-9 RURAL		
	Gr.	12	Gr.	9	Gr.	9	Gr.	12	Gr.	9	Gr.	12	Gr.	9	Gr.	12	Gr.	9	Gr.	12	Gr.
R-102	.699	.732	-.009	-.041	.110	.225	.270	.229	.024	.093	.234	.139	.112	.082	.076	.020	.054	.180	.177	.200	
R-103	.686	.752	-.029	.014	.083	.158	.187	.218	.014	.092	.196	.060	.135	.177	.098	.075	.003	.043	.010	.086	
R-104	.710	.756	-.042	-.061	.130	.224	.167	.124	.047	.036	.146	.021	.093	.036	.037	.052	.056	.088	.089	.116	
R-105	.722	.739	.062	.048	.083	.167	.212	.153	.072	.132	.193	.089	.105	.086	.014	.006	.040	.102	.162	.172	
R-106	.590	.651	.332	.488	.147	.235	.208	.171	.033	.010	.206	.176	.012	.118	.021	.067	.067	.123	.125	.094	
R-107	.533	.626	.112	.215	.072	.185	.228	.163	.074	.033	.422	.330	.079	.070	-.021	.013	.072	.143	.158	.195	
R-108	.469	.508	.075	.028	.056	.200	.185	.182	-.046	-.068	.202	.195	.159	.138	.035	.009	-.046	.046	.346	.314	
R-109	.444	.484	.015	-.014	.129	.224	.151	.095	.054	.042	.107	.050	.067	.034	.070	.090	-.028	.110	.185	.184	
R-110	.409	.458	.018	-.027	.047	.208	-.024	-.034	.026	.038	.301	.226	.007	-.051	.035	.035	.026	.127	.011	.081	
R-111	.289	.350	.072	.154	-.135	.076	.050	.056	.080	-.006	.604	.431	.031	.044	-.012	-.011	.033	.183	.151	.203	
R-112	.328	.337	-.017	-.047	.133	.240	.083	.032	-.029	.034	.322	.180	.045	-.015	.016	-.015	.006	.155	.361	.457	
R-113	.415	.401	-.013	-.042	.162	.211	.184	.143	.095	.122	.151	.058	.125	.142	.047	.066	-.028	.052	.494	.542	
R-114	.407	.382	.005	-.060	.133	.207	.084	-.053	.050	.120	.223	.078	.086	.033	.034	.040	.025	-.036	.533	.660	
R-115	.568	.610	.056	.047	.076	.147	-.012	-.011	.052	.070	.092	.027	.047	.111	.121	.126	.141	.135	.200	.205	
R-131	.671	.675	-.135	-.157	.108	.137	.180	.094	.010	.189	.226	.055	.069	.098	.065	.051	.019	.042	.077	.074	
R-132	.498	.584	-.003	.026	.017	.081	.029	.040	.055	.272	.163	.032	.064	.085	.055	.038	-.001	.000	.105	.196	
R-133	.534	.468	-.105	-.041	-.008	.020	.230	.107	.111	.305	.209	.026	.082	.092	.116	.116	.079	.206	.258	.322	
R-134	.366	.355	-.029	.031	.097	.144	.148	.081	.005	.104	.227	.060	-.036	-.095	.010	-.015	.077	.085	.255	.374	
R-139	.578	.618	-.040	-.044	.059	.188	.178	.115	.100	.182	.204	-.012	-.018	-.060	.032	.081	.054	.137	.118	.224	
R-140	.446	.442	-.116	-.037	.044	.052	.040	-.038	.119	.236	.218	-.011	-.019	-.019	.134	.116	.156	.190	.097	.127	
R-142	.446	.422	.006	-.000	.074	.129	.115	.065	.004	.138	.148	.069	.059	.094	.069	-.028	-.028	.040	.113	.078	
R-145	.039	-.011	-.034	-.010	-.031	-.011	.000	.008	.013	.017	.077	.012	-.013	.023	.052	.027	.002	.009	.152	.302	
R-146	.068	.018	-.015	.052	.007	.037	.024	.008	-.017	.038	.001	.041	.051	.052	.031	-.054	-.024	.051	.048	.038	
R-147	.542	.543	.000	.097	.135	.177	.036	-.077	.044	.244	.202	.123	.085	.135	.104	.028	.100	-.007	.136	.135	
R-150	.674	.744	-.142	-.082	.048	.070	.188	.078	.062	.272	.132	-.065	.062	.026	.084	.049	.048	.069	.020	.036	
R-162	.662	.660	-.107	-.087	.076	.130	.255	.167	.051	.318	.204	.017	.054	.004	.043	-.059	.051	.061	.161	.224	
R-212	.351	.347	.057	.087	.119	.134	.267	.151	.052	.136	.145	.028	.008	.150	.067	.020	.035	.185	.100	.038	
R-220	.448	.385	-.063	-.032	.162	.156	.490	.494	.030	-.086	.091	.162	.088	.373	.282	.164	.086	.236	.032	-.047	
R-231	.348	.318	.053	.051	.017	.010	.642	.536	.175	.221	.059	.012	-.018	.377	.090	.053	.107	.202	.069	.036	
R-232	.266	.190	-.031	-.021	.090	.080	.277	.057	.688	.389	.151	.083	-.086	.575	.152	.169	.022	.133	.034	.093	
R-233	.444	.408	.208	.177	.223	.286	.458	.371	.310	.262	.100	.075	.087	.440	.104	.101	.025	.111	.124	.092	
R-234	.369	.340	-.010	.086	.118	.127	.393	.287	.298	.272	.116	.102	.107	.529	.088	.066	.003	.057	.061	.076	
R-235	.348	.350	.008	.015	.143	.221	.292	.176	.270	.203	.121	.117	.128	.451	.014	.005	-.038	.128	.043	.111	
R-240	.463	.453	.297	.249	.207	.275	.381	.413	.087	.090	.094	.023	.062	.057	.147	.066	.022	.143	.070	.166	
R-250	.644	.605	.010	-.006	.186	.241	.331	.248	.124	.148	.169	.055	.118	.188	.150	.007	.001	.128	.149	.178	
R-260	.446	.408	.038	.027	.223	.300	.211	.247	.025	-.027	.228	.196	.011	.217	.139	.022	-.028	.132	.168	.168	
R-270	.334	.299	.115	.136	.484	.575	.054	.099	.077	.050	.352	.354	.065	.211	.104	-.025	-.032	.073	.104	.078	
R-281	.197	.104	.008	-.020	.553	.587	.057	.078	.092	.020	.223	.352	.020	.088	.226	.072	.088	.196	.057	.044	
R-282	.236	.199	.120	.116	.580	.616	.056	.091	.068	-.006	.273	.276	-.019	.042	.113	.029	.008	.170	.094	.188	
R-290	.360	.361	.102	.105	.469	.494	.185	.077	.162	.268	.187	.132	.029	.136	.162	.089	.069	.130	.058	.109	
R-311	.476	.506	.233	.289	.264	.305	.313	.210	.178	.188	.216	.123	-.011	.147	.001	.029	.048	.092	.168	.206	
R-312	.519	.551	.369	.588	.231	.190	.293	.168	.175	.113	.189	.129	-.046	.178	.069	.110	.100	.081	.097	.095	
R-333	.130	.458	.129	.629	.000	.120	.013	.062	.013	.029	.112	.137	-.044	.125	.008	.039	.011	.036	.007	-.032	
F-410	.211	.232	.096	.154	.074	.096	.273	.129	.255	.245	.154	.058	.073	.111	.168	.178	.269	.494	.152	.142	
F-420	.100	.125	.001	.035	.095	.098	.040	.032	.173	.112	.055	.054	.033	.033	.521	.148	.130	.720	.076	-.013	
F-430	.114	.125	.068	.098	.055	.069	.234	.232	.012	-.042	.035	.163	.105	.238	.570	.170	.110	.514	-.028	-.070	
F-440	.086	.078	.001	-.065	.261	.315	.009	.076	-.012	-.052	.076	.108	.011	.035	.600	.208	.184	.624	.069	.083	
P*801	.562	.030	.030		.139		-.018		-.007		.064		.093		.098		.029			-.105	
ZB ²	20.65		2.04		4.51		4.19		2.41		3.12		2.06		1.74		2.36		3.52		

TABLE 4A. Distribution of correlation residuals
after extraction of 17 principal factors

Resid. (Lower bound of interval)	Off-diag. residuals						Diag.resids.	
	non-corresp.		corresp.		all		M	F
	M	F	M	F	M	F		
.20+							49	56
.19							12	7
.18							8	5
.17							5	10
.16							3	5
.15							9	3
.14							2	2
.13							3	2
.12							4	3
.11								
.10				1		1		
.09			1	2	1	2		1
.08	1		1	2	2	2		1
.07		1	1	5	1	6		
.06	10	8	6	3	16	11		
.05	15	12	5	6	20	18		
.04	25	60	9	4	34	64		
.03	97	139	6	10	103	149		
.02	291	284	6	4	297	288		
.01	619	539	2	3	621	542		
.00	929	923	2	3	931	926		
-.01	965	980	3	1	968	981		
-.02	780	741	2	1	782	742		
-.03	404	397	1	1	405	398		
-.04	159	196			159	196		
-.05	70	88			70	88		
-.06	27	32			27	32		
-.07	15	10	1		16	10		
-.08	8	4	1	1	9	5		
-.09		2				2		
-.10	2	2			2	2		
-.11								
-.12								
-.13	1				1			
No.	4418	4418	47	47	4465	4465	95	95
Mean	-.003	-.003	.032	.041	-.002	-.002	.214	.208
S.D.	.019	.020	.034	.034	.019	.020	.221	.212

TABLE 4B. Final distribution of correlation residuals,
after extraction of 17 principal factors
and 23 test-specific factors

Resid. (Lower bound of interval)	Off-diag. residuals						Diag. resid.	
	non-corresp.		corresp.		all		M	F
	M	F	M	F	M	F		
.20+							49	56
.19							12	7
.18							8	5
.17							5	10
.16							3	5
.15							9	3
.14							2	2
.13							3	2
.12							4	3
.11								
.10								
.09								1
.08	1				1			1
.07		1				1		
.06	10	8			10	8		
.05	15	12			15	12		
.04	25	60			25	60		
.03	97	139	6	10	103	149		
.02	291	284	6	4	297	288		
.01	619	539	2	3	621	542		
.00	929	923	14	15	943	938		
-.01	965	980	14	12	979	992		
-.02	780	741	2	1	782	742		
-.03	404	397	1	1	405	398		
-.04	159	196			159	196		
-.05	70	88			70	88		
-.06	27	32			27	32		
-.07	15	10	1		16	10		
-.08	8	4	1	1	9	5		
-.09		2				2		
-.10	2	2			2	2		
-.11								
-.12								
-.13	1				1			
No.	4418	4418	47	47	4465	4465	95	95
Mean	-.003	-.003	.004	.008	-.003	-.002	.214	.208
S.D.	.019	.020	.022	.020	.019	.020	.221	.212

FIGURE 1. Subjective Rotation of Two Varimax Factors (V_{10} and V_5)
 (Based on factor analysis of Matrix 1A: Males)

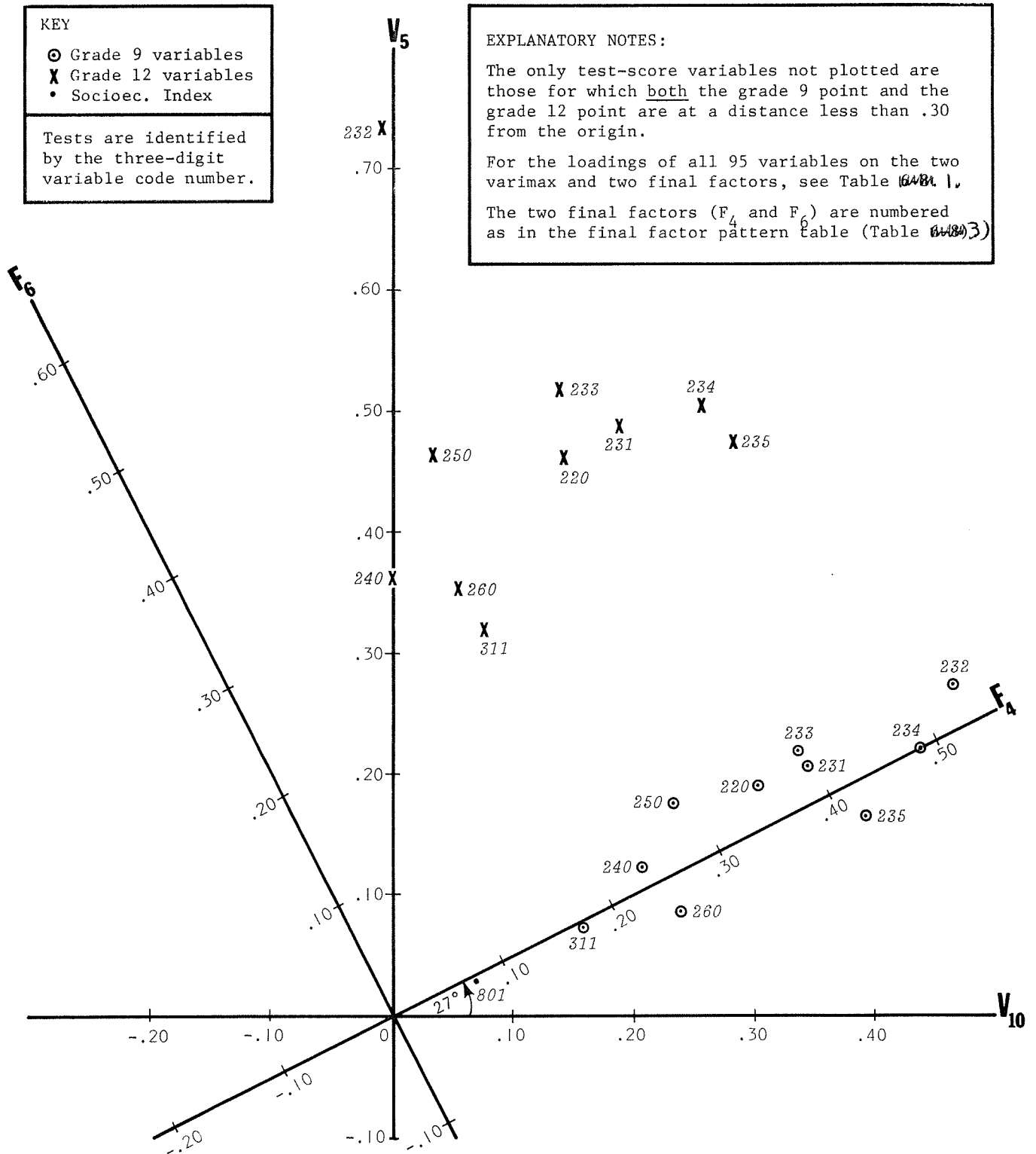


FIGURE 2. Relationship between factors F_{8a} and F_{8b}
 (Based on factor analysis of Matrix 1A: Males)

